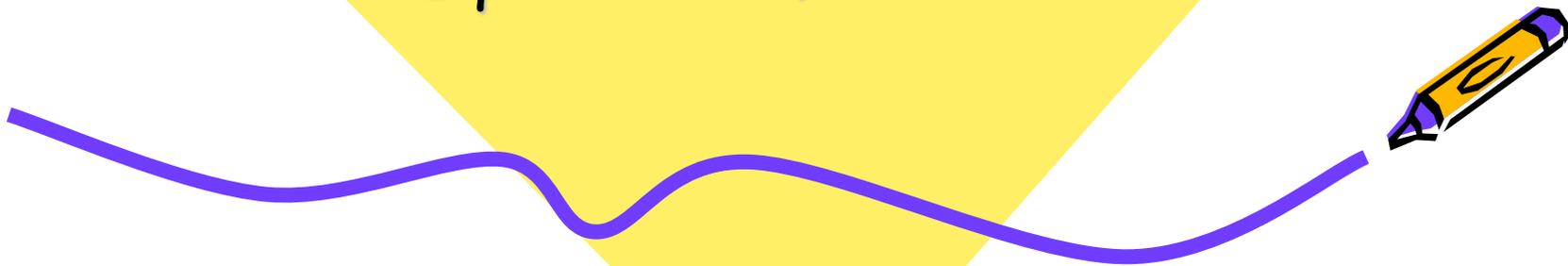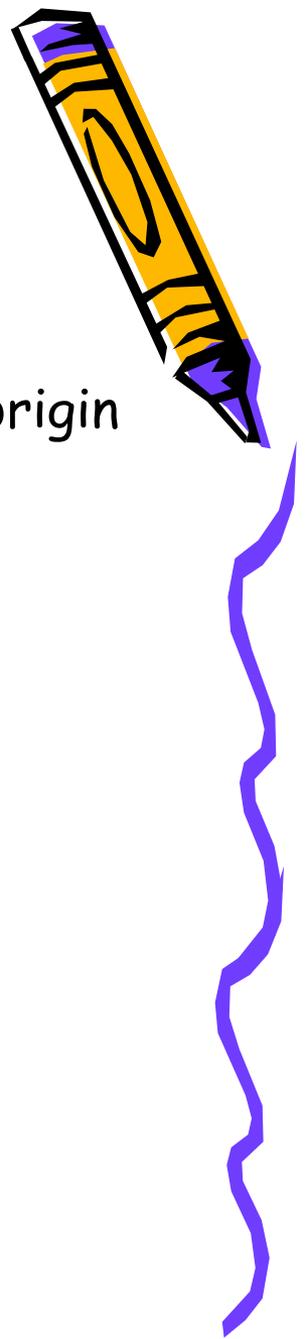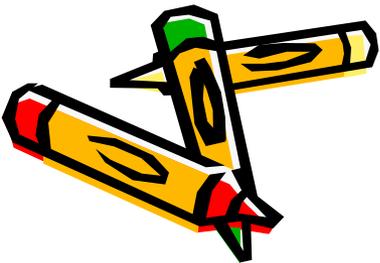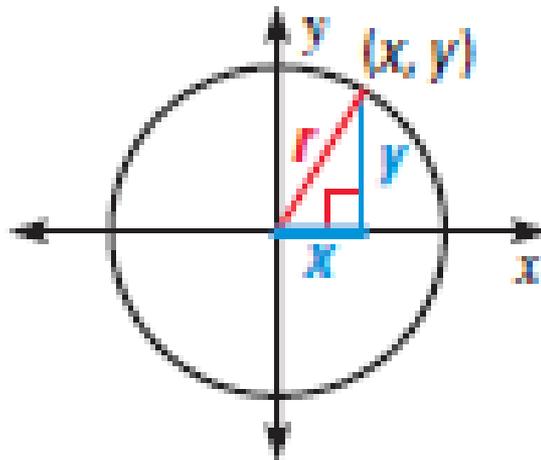# Geometry

Equations of Circles

Equations of Circles

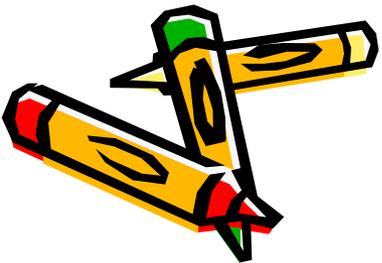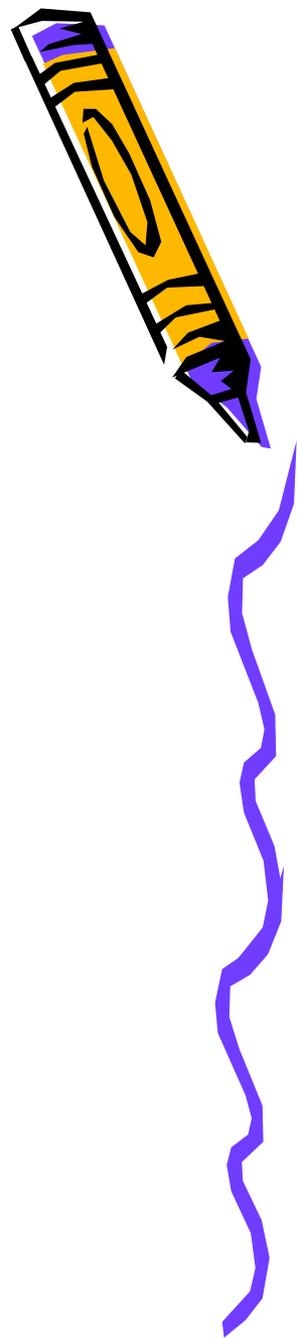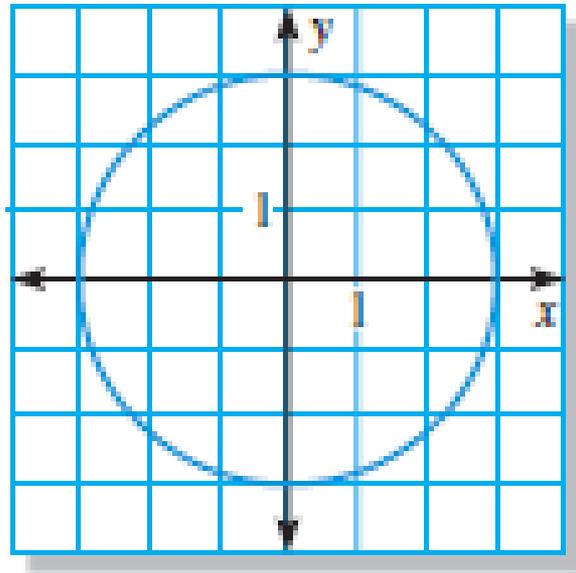Let (x,y) represent any point on a circle with center at the origin and radius r. By the Pythagorean theorem, $x^2 + y^2 = r^2$

Equations of Circles

Therefore the equation of the circle below is:
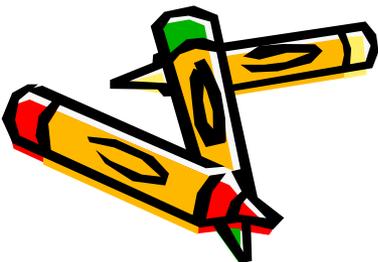$$x^2 + y^2 = 3^2 \quad or \quad x^2 + y^2 = 9$$

Equations of Circles

The Standard Form of the Equation of a Circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where h and k represent the x and y coordinates of the center of the circle respectively, and r represents the radius of the circle.
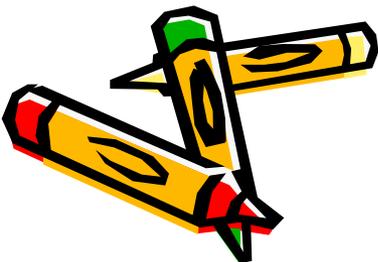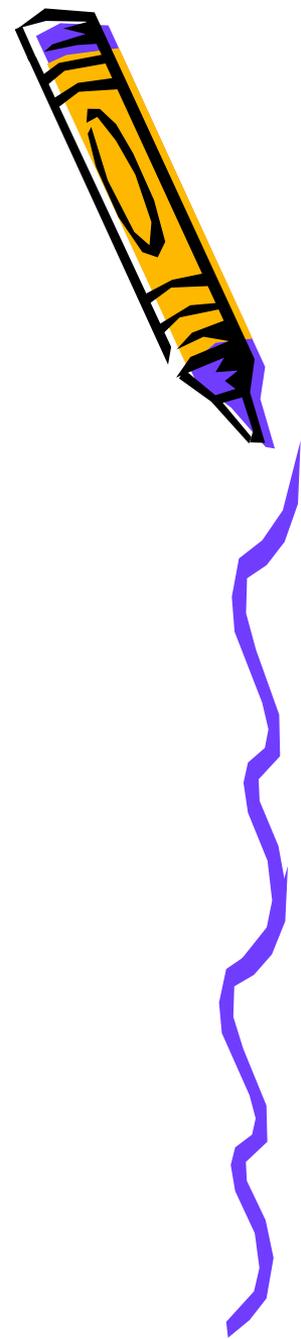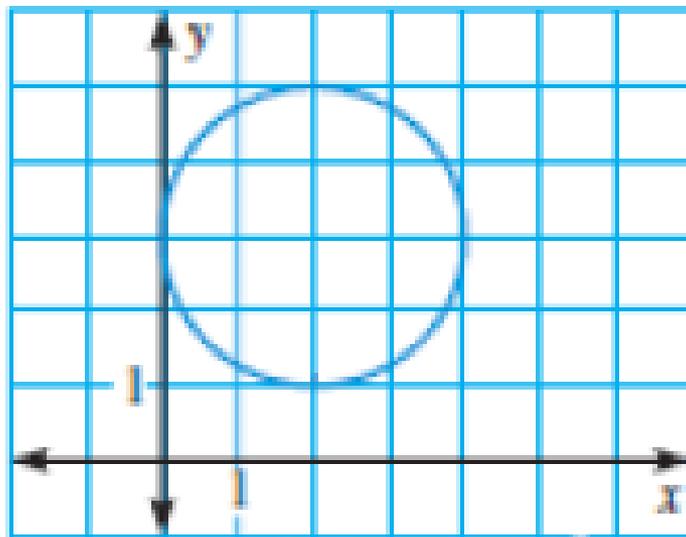
Center: (h, k)

Radius: r

Equations of Circles

Therefore the equation of the circle below is:
$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + (y - 3)^2 = 2^2$$

$$(x - 2)^2 + (y - 3)^2 = 4$$
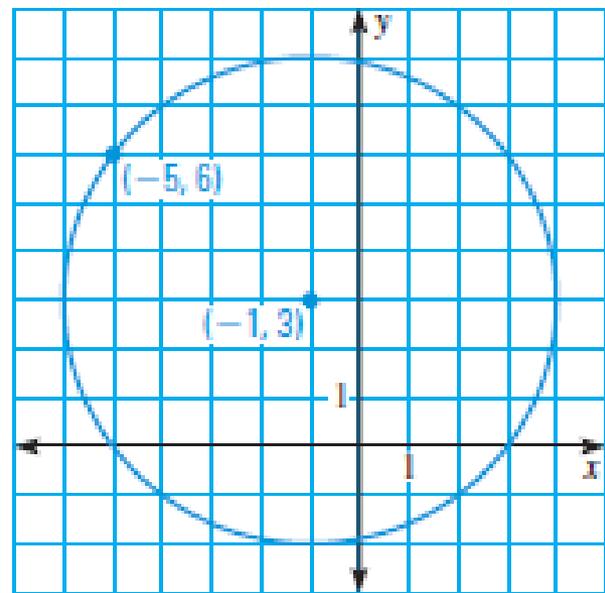
Equations of Circles

Write the equation of the circle.

Determine the center and radius: *center (-1, 3) as shown*

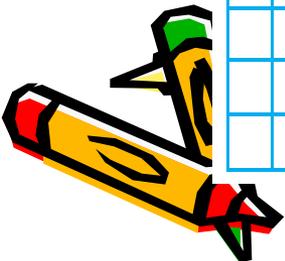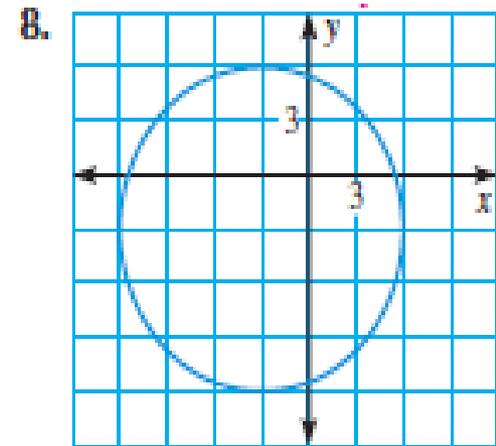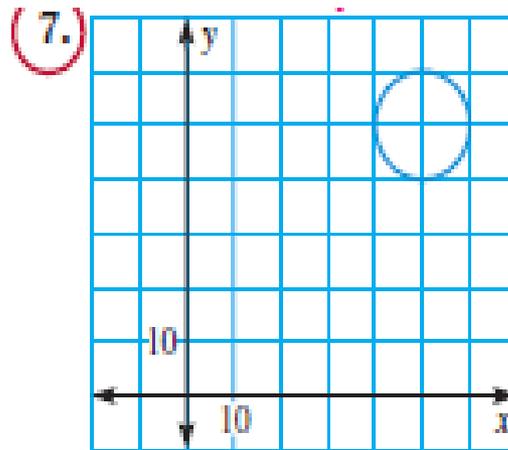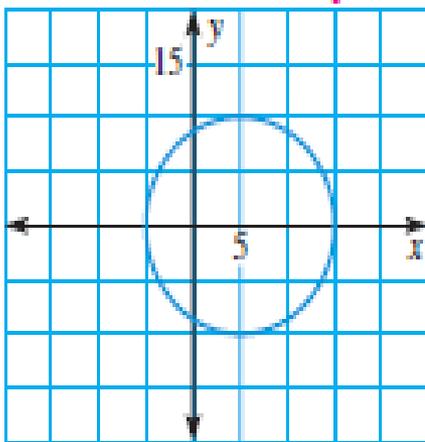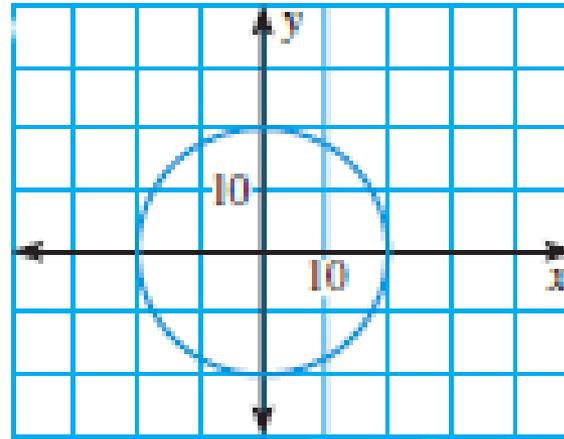*radius = 5 (use distance formula)*
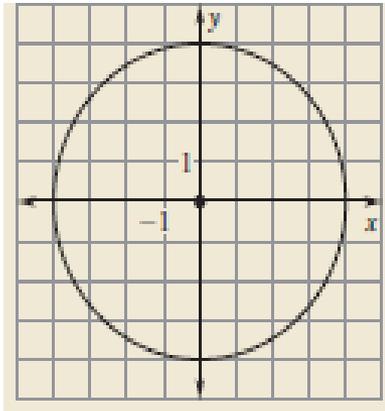
Therefore:

$$(x + 1)^2 + (y - 3)^2 = 25$$
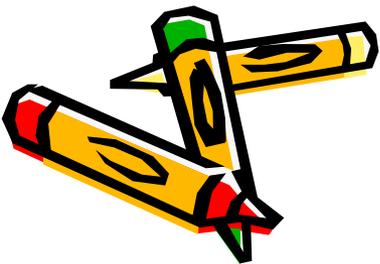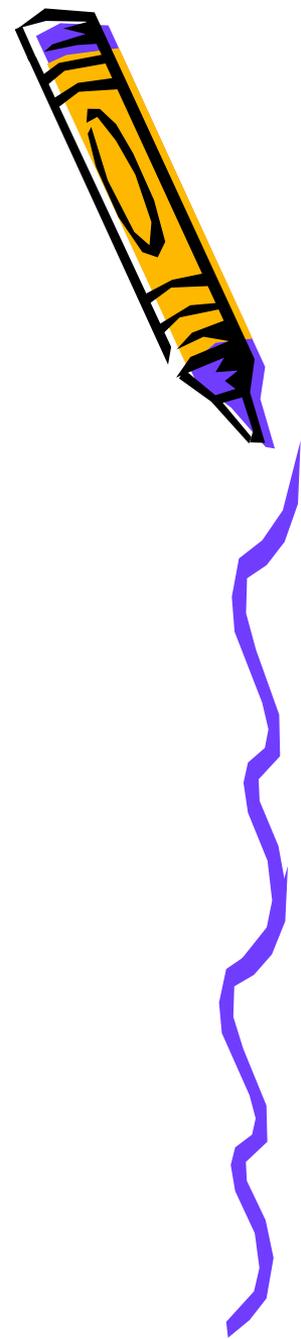
Equations of Circles

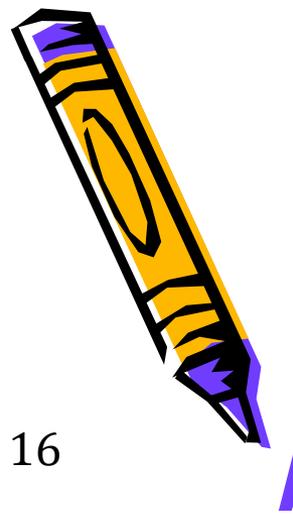Write the equation of the circle (note the scale on each graph).

7.

8.

Equations of Circles

Write the equation of the circle.

1. Center (0,-9) and Radius 4.2

2. Center (-2,3) and Radius 3.8

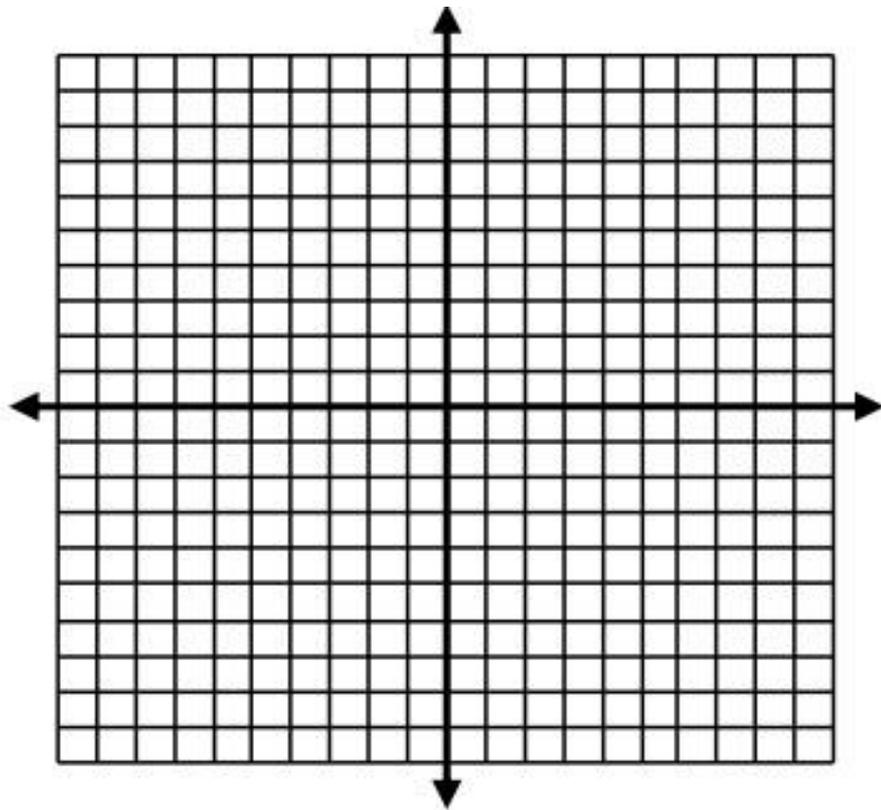3. Center (0,0) & Point on circle (0,6)

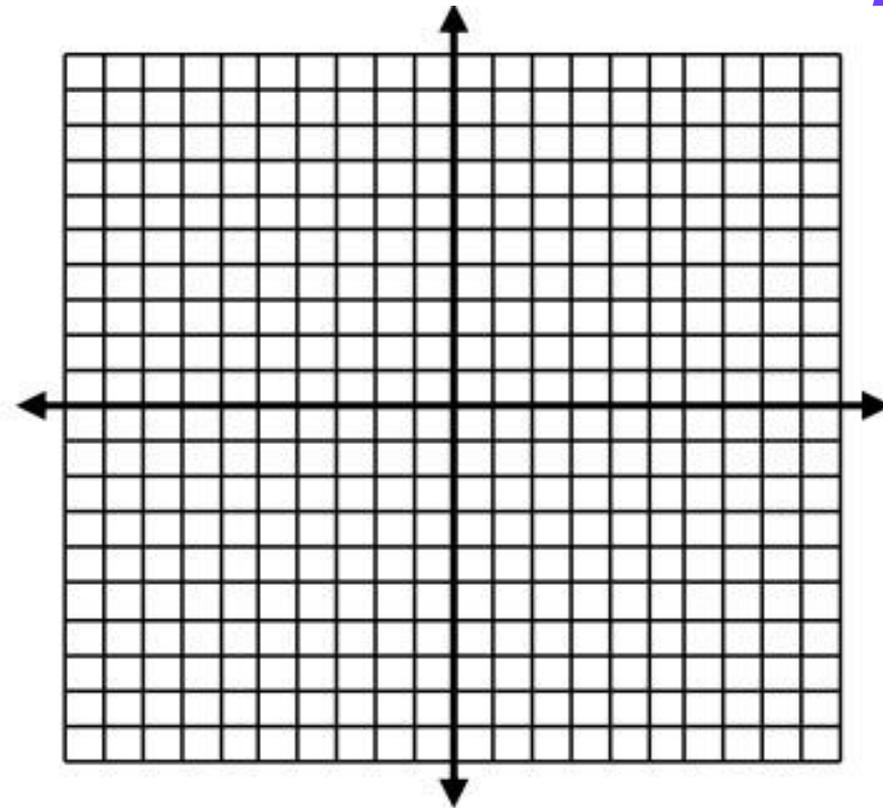4. Center (1,2) & Point on Circle (4,2)

Equations of Circles

Graph the circle.
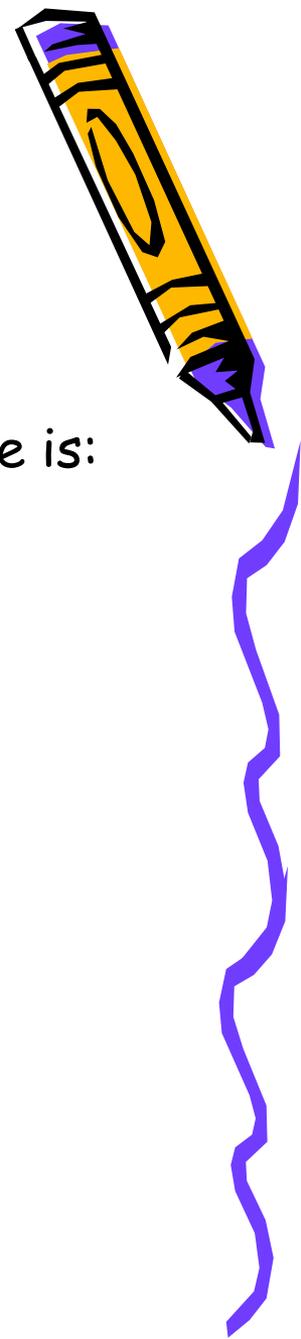
$(x+5)^2 + (y-3)^2 = 9$
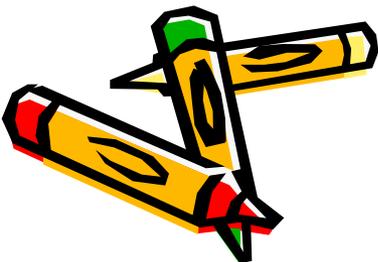
$(x-3)^2 + y^2 = 16$

Equations of Circles

An Alternate (General) Form of the Equation of a Circle is:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

In this model:

The center is: (-g, -f)

The radius is: $\sqrt{g^2 + f^2 - c}$

Equations of Circles

Using: $x^2 + y^2 + 2gx + 2fy + c = 0$

Given: $x^2 + y^2 - 6x + 4y - 12 = 0$
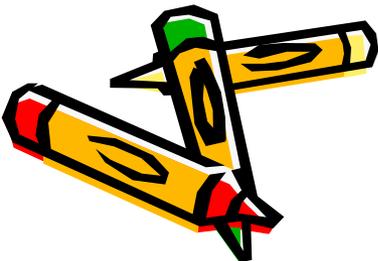
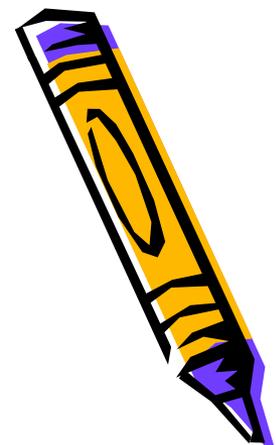Re-write it as: $x^2 + y^2 + 2(-3)x + 2(2)y - 12 = 0$

We can see that: g=-3, f=2, c=-12

Therefore:

The center is: (-g, -f) or (3, -2)

The radius is: $\sqrt{g^2 + f^2 - c}$ or $\sqrt{(-3)^2 + (2)^2 - (-12)}$ or 5
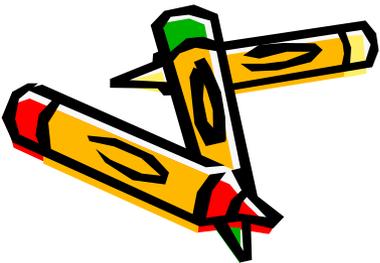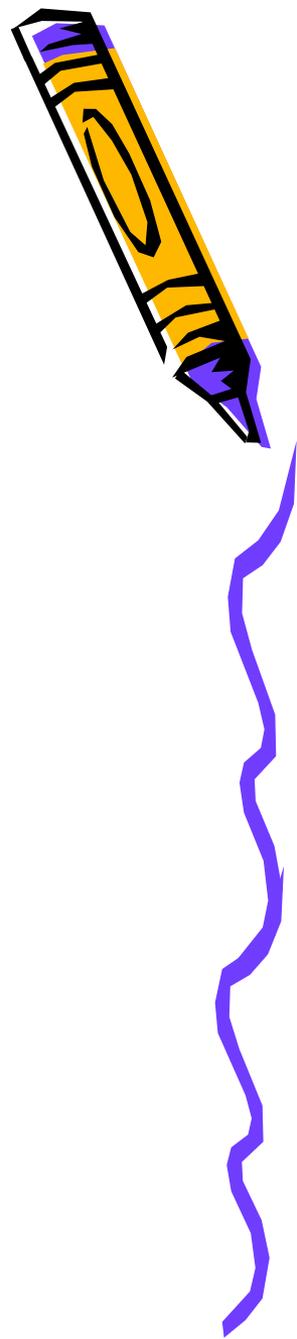
Equations of Circles

Find the radius and center of the circle given the equation:

1. $x^2 + y^2 - 4x + 6y + 4 = 0$

2. $x^2 + y^2 + 2x - 3 = 0$

Equations of Circles

Alternate approach – complete the square

Given: $x^2 + y^2 - 6x + 4y - 12 = 0$

Re-write it as: $x^2 - 6x + y^2 + 4y = 12$ (group variables and get constant to the other side)

Complete the square: $x^2 - 6x + \left(\frac{6}{2}\right)^2 + y^2 + 4y + \left(\frac{4}{2}\right)^2 = 12 + \left(\frac{6}{2}\right)^2 + \left(\frac{4}{2}\right)^2$

$x^2 - 6x + 9 + y^2 + 4y + 4 = 12 + 4 + 9$

$(x - 3)^2 + (y + 2)^2 = 12 + 9 + 4$

$(x - 3)^2 + (y + 2)^2 = 25$

The center is: (3, -2)
The radius is: 5