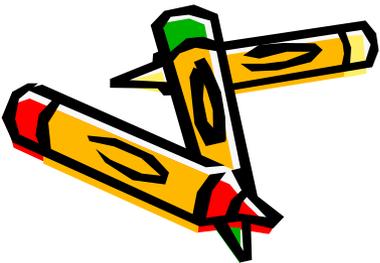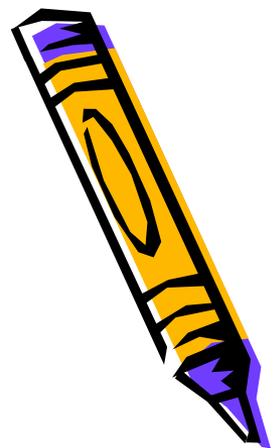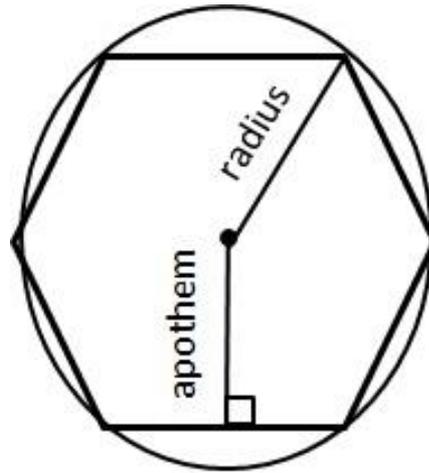# Geometry

Area of Regular Polygons

Area of Regular Polygons

A **regular polygon** is both equilateral and equiangular.

The **center** of a regular polygon is the center of the circle circumscribed around the polygon.

radius

apothem

Area of Regular Polygons

The segment from a vertex to the center is the **radius** of the circle that can be circumscribed around the polygon.
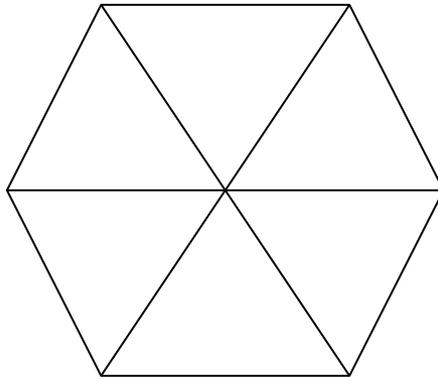
The perpendicular segment from the center to a side is an **apothem**.

radius

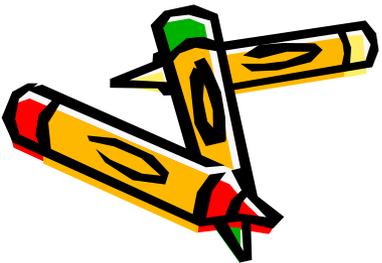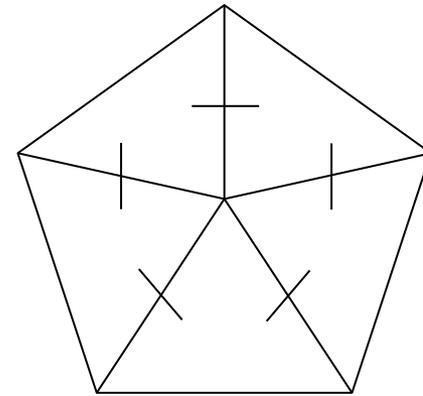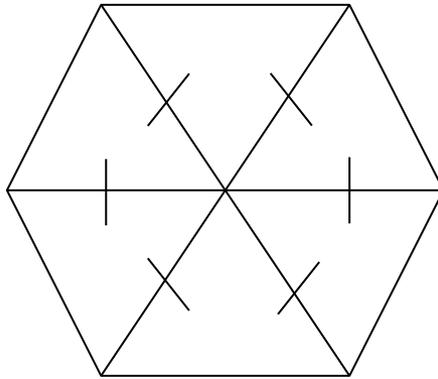apothem
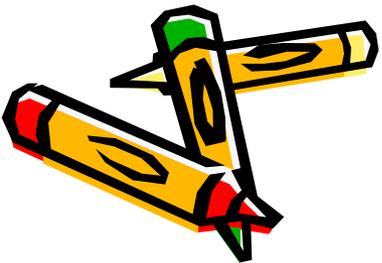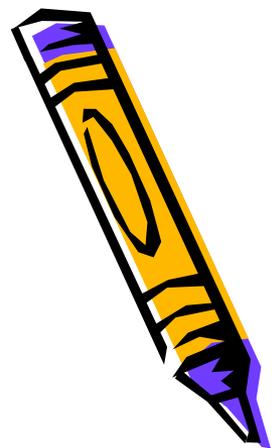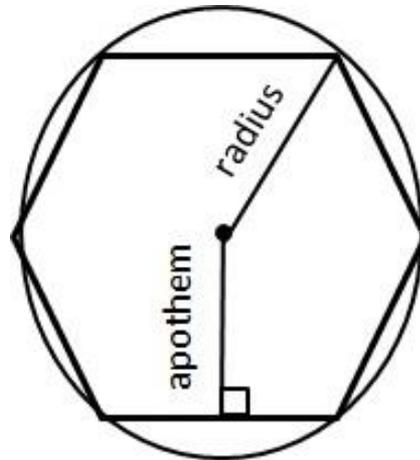
Area of Regular Polygons

Any regular polygon can be divided into congruent isosceles triangles.

The number of isosceles triangles is equal to the number of sides of the polygon.
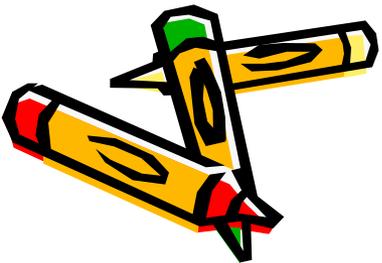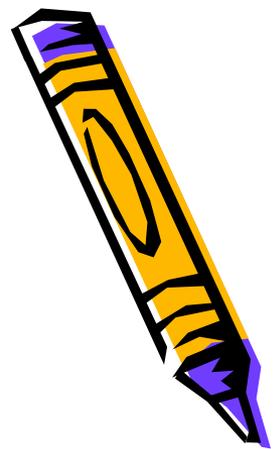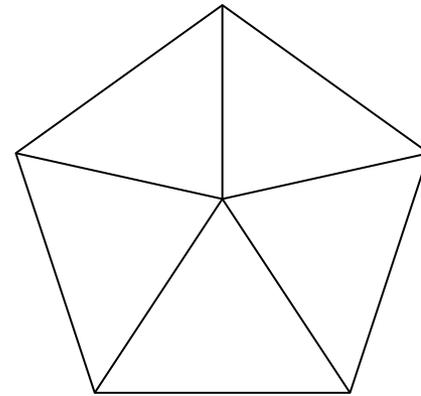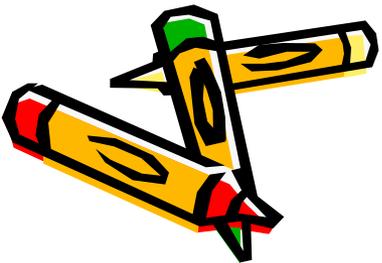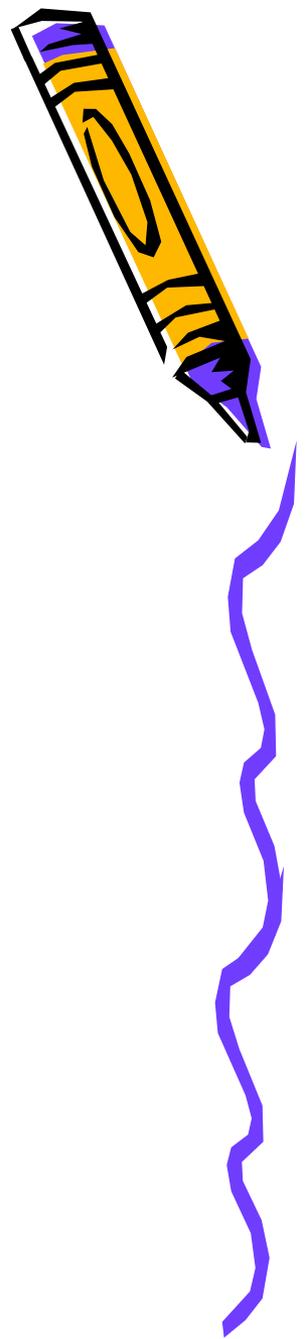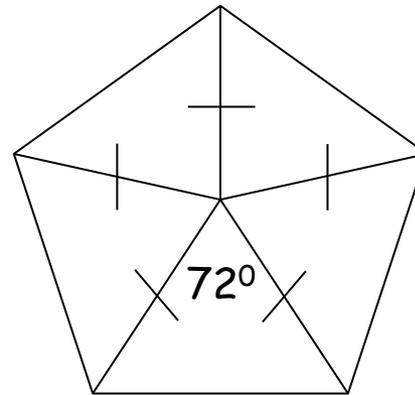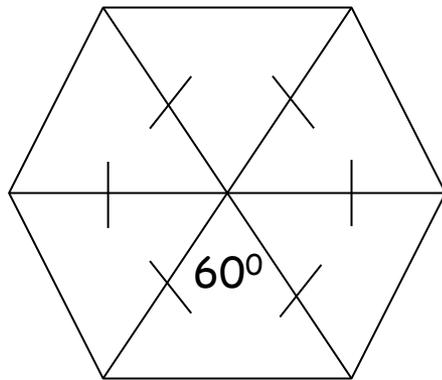
Area of Regular Polygons

All sides of the isosceles triangles are congruent as they are radii of the circumscribed circle.
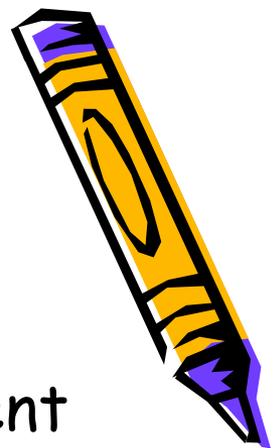
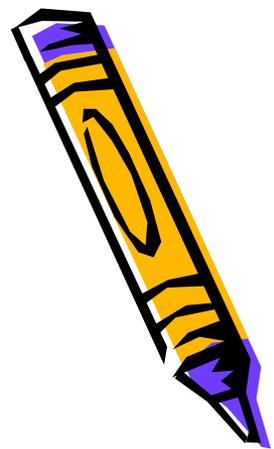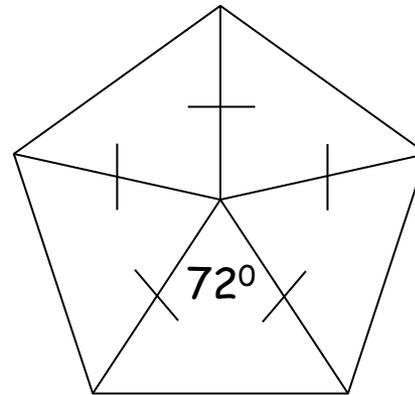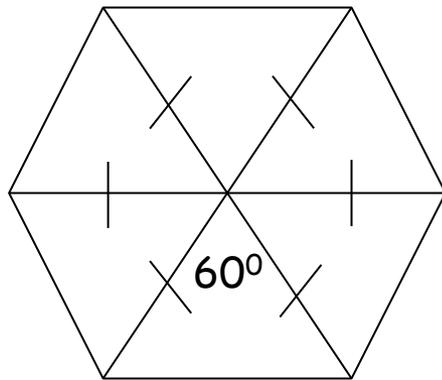# Area of Regular Polygons

The vertex angle of each isosceles triangle can be determined by dividing 360 by the number of sides of the polygon (number of triangles)

60^0

72^0

Area of Regular Polygons

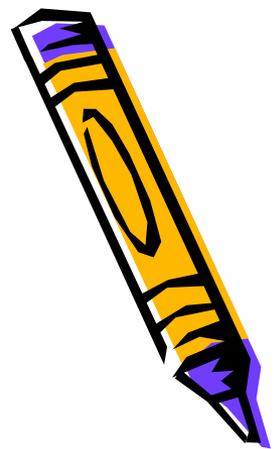Now we can use additional information provided to determine the area of an isosceles triangle and multiply by the number of triangles to the determine the area of the regular polygon.
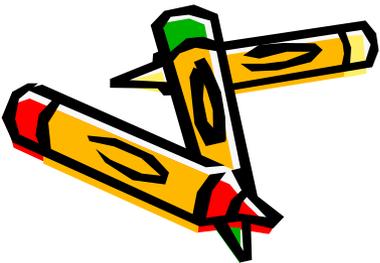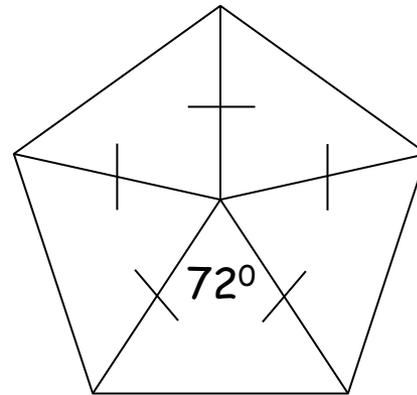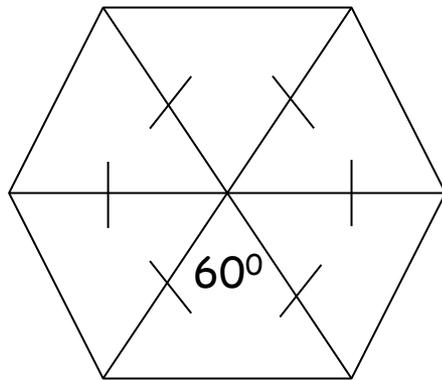
60^0

72^0
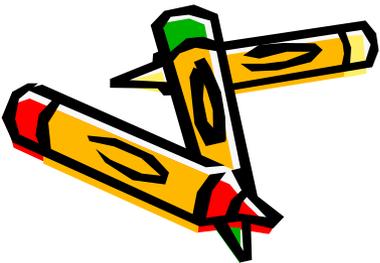
Area of Regular Polygons

Additionally, we can use the formula:

Area = $\frac{1}{2}$ (Perimeter)(apothem)   or   A = $\frac{1}{2}$ Pa

To determine the area of the regular polygon

60^0

72^0

Area of Regular Polygons

Find the area of a regular octagon inscribed in a circle with a radius of 12 cm.

1. Vertex angle = 360/8 = 45
2. The apothem of the polygon is the altitude of the isosceles triangle and bisects the vertex angle and base (side of the polygon). Use trigonometry to determine the apothem and base of the isosceles triangle:

$apothem$: $\cos 22.5 = \dfrac{a}{12}$ $a = 11.1$

$base\ /\ side$: $\sin 22.5 = \dfrac{opp}{12}$ $opp = 4.6$

times 2 so the base/side = 9.2

3. Area of the polygon:

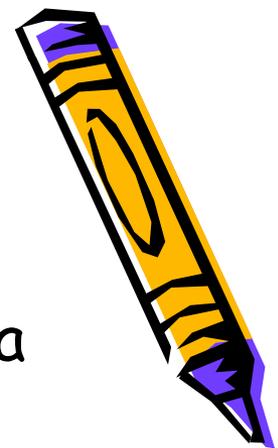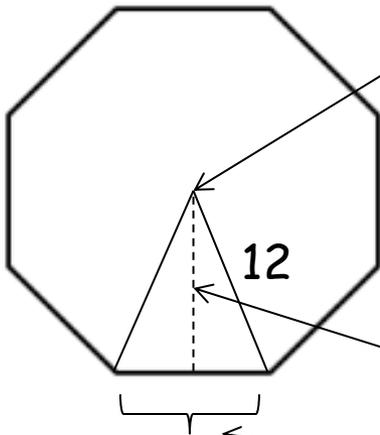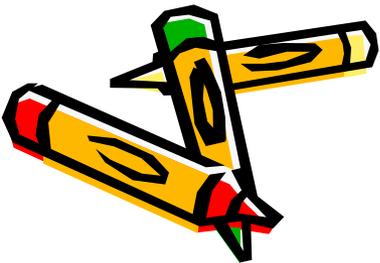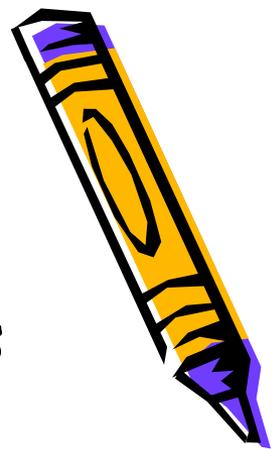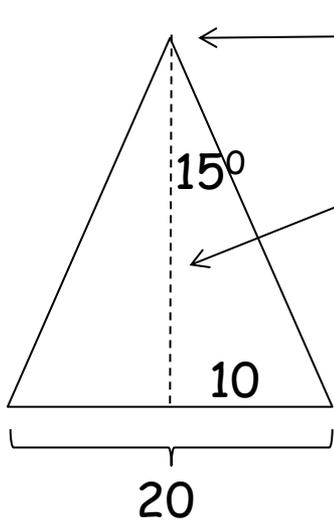$A = \dfrac{1}{2}Pa \rightarrow \dfrac{1}{2}(73.6)(11.1) = 408.5\ cm^2$

12

Area of Regular Polygons

Find the area of a regular dodecagon with sides of 20 in.

1. Vertex angle = 360/12 = 30
2. $apothem$: $\tan 15 = \dfrac{10}{a}$ $a = 37.3$
3. Area of the polygon:
   $A = \dfrac{1}{2}Pa \rightarrow \dfrac{1}{2}(240)(37.3) = 4476\ in^2$

Same as: area of triangle: $\dfrac{1}{2}(20)(37.3) = 373$
But there are 12 triangles: 373*12 = 4476